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HYPERSONIC NONVISCOUS RADIANT GAS FLOW PAST A CONE

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SUMMARY

The effect of a radiant gas on the characteristics of hypersonic flow past a cone is investigated by the "boundary layer" method developed by Freeman, Chernyy, Lyubimov and others. The problem is reduced to a nonlinear integral equation for gas enthalpy, obtained in the extreme case of the flow, and the method for its solution is indicated. This solution allows to determine the shock wave curvature at the leading edge of a cone. author

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There are many problems linked with the motion of bodies in a gas at high supersonic velocity where it is indispensable to take into account the influence of radiation heat exchange between gas particles on the parameters of the flow. The problem of hypersonic gas flow past a wedge, taking into account heat emission, was studied in the work [1], in which the author, assuming the smallness of radiation effect upon the flow characteristics, sought the solution of the problem set up in the form of series by powers of the small parameter characterizing the radiation. The question of radiation effect on flow parameters in the boundary and shock layers, and also on heat exchange in the vicinity of the critical point of a blunt body was resolved by an analogous method in the works [2], [3], [4] and [5].

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In the present work the problem of hypersonic flow past a cone by a nonviscous radiant gas is considered. For its solution the method of "boundary layer", developed in the works [6], [7] and [8], has been applied. In the extreme case of flow a nonlinear integral equation was obtained for the determination of enthalpy, and the method for its solution is indicated.

1. - PROBLEM'S EQUATIONS AND THEIR SOLUTION

We shall consider the supersonic flow past a straight circular cone of a gas. At high incident flow velocities the gas, situated in the region between the surface of the body and the shock wave, is heated to a temperature of the order of several thousand degrees. At such temperatures the radiation heat exchange between gas particles exerts a substantial effect on the parameters of the flow. For the solution of the problem set up one must in this case resort to a complete system of hydrodynamic equations, taking into account the radiant field [9], [10]. Generally speaking, such equations include the terms characterizing the density of radial energy and the radiation pressure. At gas temperature significantly below $300\,000^{\circ}$ K, the indicated terms are quite small, and they can be neglected without impairing the high degree of precision . The radiation of heated gas will exert also influence upon the parameters of the incident flow at the expense of absorption by the gas, heated in the shock layer. However, taking into account that the cold atmosphere is practically transparent for those frequencies, in which the bulk of energy is emitted by the gas heated in the shock layer to temperatures of several thousand degrees, the indicated effect can be neglected and one may approximately estimate, that the gas parameters ahead of the shock wave front are the same as at infinity.

With the above assumptions, and taking into account the radiant field, the total system of equations of gas dynamics, written in dimensionless form, will take the form [9]:

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$$
\begin{aligned}
&u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\tau\frac{\partial p}{\partial x},\\
&u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\tau\frac{\partial p}{\partial y}.\\
&\frac{\partial(\rho r u)}{dx}+\frac{\partial(\rho r v)}{dy}=0,\\
&u\frac{\partial i}{\partial x}+v\frac{\partial i}{\partial y}=\tau\left(u\frac{\partial p}{\partial x}+v\frac{\partial p}{\partial y}\right)+\xi_0 q,\\
&q=\int_0^{+\infty}\alpha_\nu d\nu\left\{\frac{1}{4\pi}\iint_{(4\pi)}J_{\nu l}d\Omega-B\nu\right\},\\
&\eta_0\frac{1}{\rho\alpha_\nu}\frac{\partial J_{\nu l}}{\partial l}+J_{\nu l}=B_\nu,\\
&r=x\sin\alpha+y\cos\alpha,\\
&\tau=\frac{1}{\rho}=\tau(p,\ i),\\
&T=T(p,\ i);\quad \alpha_\nu=\alpha_\nu(p,\ i),
\end{aligned}
\qquad (1,1)
$$

where α is the half-aperture angle of the cone; x and y are the coordinates referred to a certain characteristic dimension L, with x being counted along the cone's generatrix and y – along the normal to it; u and v are the velocity vector components, respectively along the axes x and y, referred to the velocity of the incident flow V^0; ρ is the density of the gas, referred to that of the gas in incident flow ρ^0; p is the pressure in the flow, referred to $\rho^0 V^{0^2}$; i is the enthalpy, referred to V^{0^2}; q is the rate of heat inflow at the expense of radiation to the unit of gas mass; $J_{\nu l}$ is the emission intensity of the frequency ν, propagating in the direction l; α is the mass absorption coefficient; B_ν is a Plank function.

The quantities $J_{\nu l}$ and B_ν are referred to $B_1=\frac{\sigma}{\pi}T_1^4$, and q, α_ν — to $4\pi\alpha_1 B_1$, α_1 respectively, where T_1 is the characteristic temperature; α_1 is the characteristic absorption coefficient.

Besides, it is here postulated

$$\xi_0=\frac{4\pi\alpha_1 B_1 L}{V^{0^2}};\quad \eta_0=\frac{1}{\rho^0\alpha_1 L}.$$

For the solution of the problem set up, we shall take advantage of the "boundary layer" method, developed in a series of works [6], [7] and [8]. At the same time, we shall limit ourselves in the present work to finding only the limit solution, that is, we shall seek the solution in the assumption, that the compression of gas behind the jump is infinite, or that $\tau = 0$.

We shall introduce the new independent variables

$$x \quad \text{and} \quad z = \frac{\psi}{\frac{1}{2} r^2},$$

where ψ is the current function, determined by the equations

$$\psi_x = -\rho r v, \quad \psi_y = \rho r u$$

and the existence of which is assured by the continuity equation. Then, as is not difficult to see, we shall have at the surface of the cone $z = 0$, at the shock wave front $z = 1$, and in variables (x, z) the region of flow between the shock wave and the cone surface will have the shape of a semi-infinite rectangular band.

The transition formulas to new variables may be written in the form

$$\begin{gathered}\frac{\partial}{\partial x} = \frac{\partial}{\partial x} - 2\,\frac{v + b\tau z}{r\tau}\,\frac{\partial}{\partial z},\\ \frac{\partial}{\partial y} = 2\,\frac{u - a\tau z}{r\tau}\,\frac{\partial}{\partial z},\\ a = \cos\alpha, \quad b = \sin\alpha.\end{gathered} \tag{1.2}$$

For the determination of y we have the following equation

$$2\frac{\partial y}{\partial z} = \frac{\tau r}{u - a\tau z}.$$

In the new variables (x, z) and in the extreme case, the energy conservation equations and the radial transfer equations will take the form

$$\begin{gathered}ru\frac{\partial I}{\partial x} - 2buz\,\frac{\partial i}{\partial z} = \xi_0 rq,\\ \frac{2\eta_0 u}{b}\cos(l, \hat{}\, y)\frac{\partial J_{\nu l}}{x a_\nu \partial z} + J_{\nu l} = B_\nu, \quad r = bx.\end{gathered} \tag{1,3}$$

In the extreme case the quantities u, v, p, y are determined by the equalities

$$u = \cos\alpha, \quad p = \sin^2\alpha, \quad y = 0, \quad v = 0.$$

Let us write out the boundary conditions for the system of equations (1.3):

a) we shall estimate that the cone surface temperature is given and constant

$$T = T_{w} = \text{const}.$$

Then, in the assumption that the cone emits as an absolute blackbody, we shall have

$$J^{+}_{\nu l}=B_{\nu w}(T_{w})\quad 0<\theta<\frac{\pi}{2}, \tag{1,4}$$

where θ is the angle between the directions l and that of the axis y.

The boundary condition on the surface of a nonblack cone, replacing (1.4), will be obtained below.

b) Shock wave front : $z=1$. In the extreme case we have

$$i=\frac{1}{2}\sin^2\alpha=N_0. \tag{1,5}$$

If we neglect the emission from the incident flow, the correlation

$$J^{-}_{\nu l}=0\quad \frac{\pi}{2}<\theta\leqslant\pi,$$

where $J^{-}_{\nu l}$ — the intensity of the entering radiation, will be valid at the shock wave front.

It is easy to see that the first equation of the system (1.3) with the boundary condition (1.5), is equivalent to the following :

$$i=N_0+\frac{\xi_0}{u}x\int\limits_{\sqrt{z}}^{1}q\left(x\zeta,\ \frac{z}{\zeta^2}\right)d\zeta. \tag{1,6}$$

Integrating the equation for the radial transfer, we shall arrive at the expressions

$$J^{+}_{\nu l}=e^{-\mu_\nu\sec\theta}\left(B_{\nu w}+\sec\theta\int\limits_{0}^{\mu_\nu}B_\nu e^{\mu_\nu\sec\theta}d\mu_\nu\right)\quad 0<\theta<\frac{\pi}{2},$$

$$J^{-}_{\nu l}=\sec\theta e^{-\mu_\nu\sec\theta}\int\limits_{\mu_\nu}^{\bar{\mu}_\nu}B_\nu e^{\mu_\nu\sec\theta}d\mu_\nu\quad \frac{\pi}{2}\leqslant\theta\leqslant\pi; \tag{1,7}$$

$$\mu_\nu=\frac{\mathrm{tg}\,\alpha}{2\eta_0}x\int\limits_0^z\alpha_\nu dz,\quad \bar{\mu}_\nu=\frac{\mathrm{tg}\,\alpha}{2\eta_0}x\int\limits_0^1\alpha_\nu dz.$$

Substituting (1.7) into the expression for q, we shall obtain:

$$i(x,\ z)=N_0+\xi_1 x\int\limits_{\sqrt{z}}^{1}q\left(x\zeta,\ \frac{z}{\zeta^2}\right)d\zeta,$$

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$$q=\frac{1}{2}\int_{0}^{+\infty}\alpha_\nu d\nu\left\{B_{\nu w}E_2(\mu_\nu)+\int_0^{\mu_\nu}B_\nu E_1(\mu_\nu-t)dt+\int_{\mu_\nu}^{\bar{\mu}_\nu}B_\nu E_1(t-\mu_\nu)dt-2B_\nu\right\}, \qquad (1,8)$$

$$\xi_1=\xi_0\sec\alpha.$$

Here it is postulated

$$E_1(x)=\int_1^{+\infty}\frac{e^{-xt}}{t}dt,\quad E_2(x)=\int_1^{+\infty}\frac{e^{-xt}}{t^2}dt.$$

It is easy to demonstrate, that for the functions $E_1(x)$ and $E_2(x)$ the expressions

$$E_1(x)=-\ln x-C-\sum_{k=1}^{\infty}(-1)^k\frac{x^k}{k\cdot k!},$$

$$E_2(x)=1+x\ln x+(C-1)x+\sum_{k=2}^{\infty}(-1)^{k-1}\frac{x^k}{(k-1)\cdot k!},$$

where C is the Eiler constant, are valid.

Taking into account the form of representations for the functions $E_1(x)$ and $E_2(x)$, we shall seek the solution of (1.8) in the vicinity of the point $x = 0$ in the form:

$$i=a_0+\sum_{k=1}^{\infty}\left(\sum_{m=0}^{k-1}a_{km}(z)\ln^m x\right)\cdot x^k,$$

$$B_\nu=b_0^{(\nu)}+\sum_{k=1}^{\infty}\left(\sum_{m=0}^{k-1}b_{km}^{(\nu)}(z)\ln^m x\right)\cdot x^k, \qquad (1,9)$$

$$\alpha_\nu=\beta_0^{(\nu)}+\sum_{k=1}^{\infty}\left(\sum_{m=0}^{k-1}\beta_{km}^{(\nu)}(z)\ln^m x\right)\cdot x^k.$$

The coefficients a_{km} are determined at the substitution of the the expressions (1.9) into the equation (1.8). For the first four coefficients we obtain the following expressions:

$$a_0=N_0=\text{const},$$

$$a_{10}=\frac{1}{2}\xi_1\left[\int_0^{+\infty}\beta_0^{(\nu)}(B_{\nu w}-2b_0^{(\nu)})d\nu\right](1-\sqrt{z}),$$

$$a_{21}=-\frac{1}{4}h\xi_1\left\{2(1-z)\int_0^{+\infty}\beta_0^{(\nu)}b_0^{(\nu)}d\nu+z\ln z\int_0^{+\infty}\beta_0^{(\nu)2}B_{\nu w}d\nu\right\},$$

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$$a_{20}^{1}=\frac{1}{2}\xi_1\left\{\frac{1}{4}\xi_1(1-\sqrt{z})^2\left[\int_0^{+\infty}\beta_0^{(\nu)}(B_{\nu w}-2b_0^{(\nu)})\,d\nu\int_0^{+\infty}S_{10}^{(\nu)}(B_{\nu w}-2b_0^{(\nu)})\,d\nu\right]-\right.$$

$$-\frac{3}{8}hz\ln^2 z\int_0^{+\infty}B_w^{(\nu)}\beta_0^{(\nu)2}d\nu-\frac{1}{4}hz\ln z\int_0^{+\infty}B_w^{(\nu)}\beta_0^{(\nu)2}\ln\left(h\beta_0^{(\nu)}\right)d\nu-$$

$$-\frac{1}{2}(C-1)hz\ln z\int_0^{+\infty}\beta_0^{(\nu)2}b_0^{(\nu)}d\nu+h(z\ln z+3\sqrt{z}-3z)\int_0^{+\infty}\beta_0^{(\nu)}b_0^{(\nu)}d\nu-$$

$$-\frac{1}{4}h(1-z)\int_0^{+\infty}b_0^{(\nu)}\beta_0^{(\nu)2}\left[2\ln\left(h\beta_0^{(\nu)}\right)-1\right]d\nu+\frac{1}{4}z\ln z\int_0^{+\infty}b_0^{(\nu)}\beta_0^{(\nu)2}d\nu-$$

$$-h\left[(1-\sqrt{z})^2\ln(1-\sqrt{z})+(1+\sqrt{z})^2\ln(1+\sqrt{z})-\right.$$

$$\left.-3z+2\sqrt{z}(1-2\ln 2)+1\right]\int_0^{+\infty}\beta_0^{(\nu)2}b_0^{(\nu)}d\nu-\frac{1}{2}Ch(1-\sqrt{z})\int_0^{+\infty}b_0^{(\nu)}\beta_0^{(\nu)2}d\nu-$$

$$\left.-\frac{1}{2}\xi_1(1-\sqrt{z})^2\int_0^{+\infty}\beta_0^{(\nu)}(B_{\nu w}-2b_0^{(\nu)})\,d\nu\int_0^{+\infty}P_{10}^{(\nu)}\beta_0^{(\nu)}d\nu\right\}.$$

$$P_{10}=\left.\frac{\partial B_\nu}{\partial l}\right|_{l=a_0},$$

$$S_{10}^{(\nu)}=\left.\frac{\partial a_\nu}{\partial l}\right|_{l=a_0},\qquad(1{,}10)$$

$$h=\frac{\sin\alpha}{2\eta_0 u}.$$

2. - SETTING UP THE PROBLEM OF HYPERSONIC RADIANT GAS FLOW PAST A NONBLACK CONE

Let us introduce in the vicinity of a certain point M, lying on the surface of the cone, a local spherical system of coordinates (φ, θ) (Fig.1). Assume that a monochromatic radiation of frequency ν_1, propagating in the direction l_1, and whose intensity is $J_{\nu_1 l_1}$, is incident upon the surface of the cone. If we denote by ε_{ν_1} the coefficient of blackness of cone's surface for the monochromatic radiation of frequency ν_1, the quantity $(1-\varepsilon_{\nu_1})J_{\nu_1 l_1}$ will determine the aggregate intensity of the reflected radiation. At the same time, its intensity in the frequency ν and in the propagation direction l will be determined by the expression

$$\gamma_{\nu\nu_1 ll_1}(1-\varepsilon_{\nu_1})J_{\nu_1 l_1}.$$

The thus introduced function $\gamma_{\nu\nu_1 l l_1}$ will be called **reflection indicatrix.** From the determination of $\gamma_{\nu\nu_1 l l_1}$ stems the normalization condition for the reflection indicatrix

$$\int_0^{+\infty} d\nu \iint_{(2\pi)} \gamma_{\nu\nu_1 l l_1} d\Omega = 1.$$

It is now easy to obtain the boundary condition on the surface of a nonblack cone, replacing (1.4); it has the form

$$J^+_{\nu l} = \varepsilon_\nu B_{\nu w} + \int_0^{+\infty} d\nu_1 \iint_{(2\pi)} (1 - \varepsilon_{\nu_1}) \gamma_{\nu\nu_1 l l_1} J^-_{\nu_1 l_1} d\Omega_1. \qquad (2,1)$$

In this expression the first addend determines the intensity of cone surface's natural emission, and the second defines the intensity of reflected radiation.

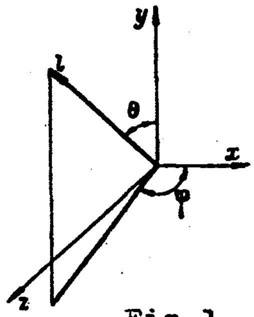

Fig. 1

The form of the function $\gamma_{\nu\nu_1 l l_1}$ is determined by the properties of surface material and of the surrounding medium. We shall subsequently assume that $\gamma_{\nu\nu_1 l l_1}$ has the form

$$\gamma_{\nu\nu_1 l l_1} = \varkappa_{\nu\nu_1} \omega_{l l_1},$$

where $\varkappa_{\nu\nu_1}$ is the frequency reflection indicatrix; $\omega_{l l_1}$ is the angular reflection indicatrix.

Let us consider two extreme cases :

1.- Specular reflection, when

$$\omega_{l l_1} = \delta(\varphi_1 - \varphi)\, \delta[\theta_1 - (\pi - \theta)],$$

where $\delta(\varphi_1 - \varphi)$, $\delta[\theta_1 - (\pi - \theta)]$ are Dirak functions, such that

$$\iint_{(2\pi)} \delta(\varphi_1 - \varphi)\, \delta[\theta_1 - (\pi - \theta)]\, d\Omega_1 = 1.$$

In this case (2.1) will take the form

$$J^+_{\nu l} = \varepsilon_\nu B_{\nu w} + \int_0^{+\infty} (1 - \varepsilon_{\nu_1}) \varkappa_{\nu\nu_1} J^-_{\nu_1 l_1} d\nu_1. \qquad (2,2)$$

2.- The surface of the cone scatters the incident radiation identically in all directions (diffusive reflection).- Then $\omega_{l l_1} = \frac{1}{2\pi}$, and

$$J^+_{\nu l} = \varepsilon_\nu B_{\nu w} + \frac{1}{2\pi} \int_0^{+\infty} \varkappa_{\nu\nu_1} (1 - \varepsilon_{\nu_1})\, d\nu_1 \iint_{(2\pi)} J^-_{\nu_1 l_1} d\Omega_1. \qquad (2,3)$$

For $\varkappa_{\nu\nu_1}$ the correlation

$$\int_0^{+\infty} \varkappa_{\nu\nu_1} d\nu = 1.$$

is valid in both cases. If we assume that the reflection from the cone surface takes place with no frequency variation, then

$$\varkappa_{\nu\nu_1} = \delta(\nu_1 - \nu),$$

$$J_{\nu l}^{+}(\varphi, \theta) = \varepsilon_\nu B_{\nu w} + (1 - \varepsilon_\nu) J_{\nu l_1}^{-}(\varphi, \pi - \theta), \qquad (2,2a)$$

$$J_{\nu l}^{+}(\varphi, \theta) = \varepsilon_\nu B_{\nu w} + \frac{1 - \varepsilon_\nu}{2\pi} \iint\limits_{(2\pi)} J_{\nu l_1}^{-} d\Omega_1. \qquad (2,3a)$$

Subsequently, for the simplification of operations, the boundary condition on cone surface will be written in the form (2.2 a) or (2.3 a).

Integrating the radial transfer equation, taking into account (2.2a) or (2.3a), and substituting the result obtained into the expression for q, we shall have :

— for the specular reflection

$$q = \frac{1}{2} \int_0^{+\infty} a_\nu d\nu \left\{ \varepsilon_\nu B_{\nu w} E_2(\mu_\nu) + (1 - \varepsilon_\nu) \int_0^{\mu_\nu} B_\nu E_1(t + \mu_\nu) dt + \right.$$

$$\left. + \int_0^{\mu_\nu} B_\nu E_1(\mu_\nu - t) dt + \int_{\mu_\nu}^{\bar{\mu}_\nu} B_\nu E_1(t - \mu_\nu) dt - 2B_\nu \right\} \qquad (2,4)$$

— for the diffusive reflection

$$q = \frac{1}{2} \int_0^{+\infty} a_\nu d\nu \left\{ \varepsilon_\nu B_{\nu w} E_2(\mu_\nu) + (1 - \varepsilon_\nu) E_2(\mu_\nu) \int_0^{\mu_\nu} B_\nu E_1(t) dt + \right.$$

$$\left. + \int_0^{\mu_\nu} B_\nu E_1(\mu_\nu - t) dt + \int_{\mu_\nu}^{\mu_\nu} B_\nu E_1(t - \mu_\nu) dt - 2B_\nu \right\} \qquad (2,5)$$

Analogous expressions for the case of diffusive reflection are brought up in [11].

The equality (1.6) presents in conjunction with (2.4) or (2.5) the integral equation for the determination of enthalpy. The solution of the equations obtained may be sought for anew in the form (1.9). For the coefficients of expansions we shall have expressions, analogous to (1.10), which we shall omit here on account of the cumbersomeness.

As an example, illustrating the influence of radiation upon the gas parameters in the shock layer, we shall consider the motion of a right circular cone with a half-aperture angle $\alpha = 35^{\circ}$ in the terrestrial atmosphere at 25 km altitude and with a Mach number M = 30. The computation was conducted with the utilization of the air absorption factor and of the degrre of body surface blackness ϵ, averaged by frequency. At the same time, it was assumed that the averaged absorption factor of the gas is not dependent on temperature.

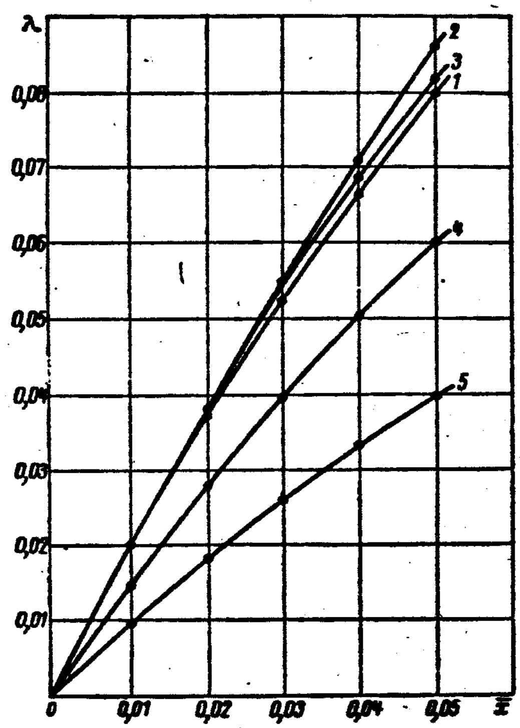

Fig 2.- $1-\epsilon=0$; $2-\epsilon=0{,}5$, $T_w=0$; $3-\epsilon=1$, $T_w=0$; $4-\epsilon=0{,}5$, $T_w=1$; $5-\epsilon=1$, $T_w=1$.

Plotted in Fig. 2 are the graphs of the function $\lambda = \lambda(x)$, where λ is a quantity, characterizing the influence of radiation on the gas enthalpy in body surface's abutment area :

$$\lambda = \frac{N_0 - i_0(x, 0)}{\xi_2},$$

$$\bar{x} = hx,$$

$$\xi_2 = \frac{8\pi B_1}{\rho^0 V^{0^3} \sin\alpha}.$$

We have chosen for the characteristic temperature T_1 the gas temperature at $\bar{x} = 0$. In the case under consideration $T_1 = 6\,200^{\circ}$ K. The dependences here constructed are valid in a small neighborhood of the nose cone, for which the terms of the order of $\bar{x}^2 \ln^2 \bar{x}$ and higher are neglectingly small by comparison with $\bar{x}$.

3. - SHOCK WAVE CURVATURE AT FORWARD POINT OF THE NOSE CONE

The equation (1.2) is available for the determination of y. Let us apply to it the method of consecutive approximations.

We shall take for the zero approximation the solution

$$u=u_0=\cos\alpha;\quad p=p=\sin^2\alpha;\quad r=r_0=x\sin\alpha;\quad \tau=\tau_0.$$

The quantity τ_0 will be determined from the equation of state $\tau_0=\tau(i_0, p_0)$. Then, we shall obtain for y in the first approximation

$$y_1=\frac{1}{2}x\,\mathrm{tg}\,\alpha\int\limits_0^z\frac{\tau_0}{1-\tau_0 z}dz.$$

The shape of the shock wave is determined by the equality

$$\bar{y}_1=\frac{1}{2}x\,\mathrm{tg}\,\alpha\int\limits_0^1\frac{\tau_0}{1-\tau_0 z}dz.$$

Expanding the subintegral expression into series by powers

$$x^k\ln^m x\quad\left\{\begin{array}{l}k=0,\ 1,\ 2,\ \ldots,\\ m=0,\ 1,\ 2,\ \ldots,\ k-1\end{array}\right\}$$

and integrating termwise, we shall find

$$y_1=\frac{1}{2}\mathrm{tg}\,\alpha\left\{-x\ln|1-\gamma_0|+\omega_{10}\left[\frac{1}{2\gamma_0^{3/2}}\ln\left|\frac{1+\sqrt{\gamma_0}}{1-\sqrt{\gamma_0}}\right|-\frac{1}{\gamma_0}\right]x^2+o(x^2\ln x)\right\},$$

$$\gamma_0=\tau(a_0,\ p_0),$$

$$\omega_{10}=\frac{\partial\tau}{\partial i}\bigg|_{i=a_0}\cdot\frac{1}{2}\xi_1\left\{\int\limits_0^{+\infty}\beta_0^{(\nu)}\left[B_{\nu\infty}-2b_0^{(\nu)}\right]d\nu\right\}.$$

Hence K, which is the curvature of the shock wave in the leading edge, can be easily determined as follows :

$$K=\omega_{10}\,\mathrm{tg}\,\alpha\frac{\frac{1}{2\gamma_0^{3/2}}\ln\left|\frac{1+\sqrt{\gamma_0}}{1-\sqrt{\gamma_0}}\right|-\frac{1}{\gamma_0}}{\left[1+\frac{1}{4}\mathrm{tg}^2\alpha\ln^2|1-\gamma_0|\right]^{3/2}}. \qquad (3,1)$$

At $\gamma_0\ll 1$, which takes place at sufficient high incident flow velocity, (3.1) takes the form

$$K=\omega_{10}\,\mathrm{tg}\,\alpha\left[\frac{1}{3}+\frac{1}{5}\gamma_0+o\left(\gamma_0^2\right)\right]. \qquad (3,2)$$

It follows from (3.2) that the sign of the shock wave front curvature coincides with that of the quantity ω_{10}. From here, the shock wave is convex upward if $T_w<\sqrt[4]{2}T_0(0,0)$, and concave upward if $T_w>\sqrt[4]{2}T_0(0,0)$, where $T_0(0,0)$ is the temperature of gas at nose cone front.

It follows from the above-expounded considerations, that the radiation of a heated gas in the region between the shock wave and the surface of the cone exerts first of all an influence on the temperature, heat content and density of the gas. The shock wave is distorted, and, at the same time, the direction of convexity in its leading edge depends on the correlations between the cone surface temperature and the gas temperature at the advanced point of the nose cone.

Note, that the results obtained can be easily extended to the case of hypersonic flow of radiant gas past an arbitrary peaked body of revolution.

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***** THE END *****

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